

QUANTUM GRAVITY AND MAXIMUM ATTAINABLE VELOCITIES IN THE STANDARD MODEL

Jorge Alfaro

*Facultad de Física, Pontificia Universidad Católica de Chile
Casilla 306, Santiago 22, Chile.*

Abstract.

A main difficulty in the quantization of the gravitational field is the lack of experiments that discriminate among the theories proposed to quantize gravity. Recently we showed that the Standard Model(SM) itself contains tiny Lorentz invariance violation(LIV) terms coming from QG. All terms depend on one arbitrary parameter α that set the scale of QG effects. In this talk we review the LIV for mesons nucleons and leptons and apply it to study several effects, including the GZK anomaly.

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INTRODUCTION

Recently several proposal have been advanced to select theories and predict new phenomena associated to the Quantum gravitational field [1, 2, 3]. Most of the new phenomenology is associated to some sort of Lorentz invariance violations(LIV's)[4, 5, 6]. More recently [7], this approach has been subjected to severe criticism.

In previous papers [8], we asserted that the main effect of QG is to deform the measure of integration of Feynman graphs at large four momenta by a tiny LIV. The classical lagrangian is unchanged. In a similar manner, we can say that QG deforms the metric of space-time, introducing a tiny LIV proportional to $(d-4)\alpha$, d being the dimension of space time in Dimensional Regularization and α is the only arbitrary parameter in the model. Such small LIV could be due to quantum fluctuations of the metric of space-time produced by QG:virtual black holes as suggested in[1], D-branes as in [10], compactification of extra-dimensions or spin-foam anisotropies [11]. A precise derivation of α will have to wait for additional progress in the available theories of QG¹ An intriguing possibility may be provided by the anisotropy between spatial and temporal directions found necessary to recover our universe at macroscopic scales in a recent numerical simulation of Quantum Gravity[9].

It is possible to have modified dispersion relations without a preferred frame(DSR)[12]. Notice, however, that in our case the classical lagrangian is invariant under usual linear

¹ Such derivation must explain why the LIV parameter is so small. Progress in this direction is in [3]. There α appears as $(l_P/L)^2$, where l_P is Planck's lenght and L is defined by the semiclassical gravitational state in Loop Quantum Gravity. If $L \sim 10^{11}l_P$, an α of the right order is obtained

Lorentz transformations but not under DSR. So our LIV is more similar to radiative breaking of usual Lorentz symmetry than to DSR. Moreover the regulator R defined below and the deformed metric (3) are given in a particular inertial frame, where spatial rotational symmetry is preserved. That is why, in this paper we are ascribing to the point of view of [5] which is widely used in the literature. The preferred frame is the one where the Cosmic Background Radiation is isotropic.

Within the Standard Model, such LIV implies several remarkable effects, which are wholly determined up to one arbitrary parameter (α). The main effects are:

The maximal attainable velocity for particles is not the speed of light, but depends on the specific couplings of the particles within the Standard Model. Noticeably, this LIV of the dispersion relations is the only acceptable, according to the very stringent bounds coming from the Ultra High Energy Cosmic Rays (UHECR) spectrum[13]. Moreover, the specific interactions between particles in the SM, determine different maximum attainable velocities for each particle, a necessary requirement to explain the Greisen[14], Zatsepin and Kuz'min[15](GZK) anomaly[5, 13, 16]. Since the Auger[17] experiment is expected to produce results in the near future, powerful tests of Lorentz invariance using the spectrum of UHECR will be available.

Also birefringence occurs for charged leptons, but not for gauge bosons. In particular, photons and neutrinos have different maximum attainable velocities. This could be tested in the next generation of neutrino detectors such as NUBE[20, 21].

This paper is organized as follows: In chapter 2, we present the LIV cutoff regulator; section 3 contains the effect of the regulator on One Particle Irreducible Green functions(1PI); section 4 defines LIV dimensional regularization ; Explicit one loop computations are contained in section 5; the LIV for mesons and baryons is found in chapter 6; Reactions thresholds and bounds on α are derived in section 7; Section 8 contains our conclusions.

CUTOFF REGULATOR

To see what are the implications of the asymmetry in the measure for renormalizable theories, we will represent the Lorentz asymmetry of the measure by the replacement

$$\int d^d k \rightarrow \int d^d k R\left(\frac{k^2 + \alpha k_0^2}{\Lambda^2}\right)$$

Here R is an arbitrary function, Λ is a cutoff with mass dimensions, that will go to infinity at the end of the calculation. We normalize $R(0) = 1$ to recover the original integral. $R(\infty) = 0$ to regulate the integral. α is a real parameter. Notice that we are assuming that rotational invariance in space is preserved. More general possibilities such as violation of rotational symmetry in space can be easily incorporated in our formalism.

This regulator has the property that for logarithmically divergent integrals, the divergent term is Lorentz invariant whereas when the cutoff goes to infinity a finite LIV part proportional to α remains.

ONE LOOP ANALYSIS

We study explicitly the case of bosons. A similar analysis apply for fermions and gauge bosons[8]. Let D be the naive degree of divergence of a One Particle Irreducible (1PI) graph. The change in the measure induces modifications to the primitively log divergent integrals($D=0$) In this case, the correction amounts to a finite LIV. The finite part of 1PI Green functions will not be affected. Therefore, Standard Model predictions are intact, except for the maximum attainable velocity for particles[5] and interaction vertices, which receive a finite wholly determined contribution from Quantum Gravity.

Let us analyze the primitively divergent 1PI graphs for bosons first. **Self energy:** $\chi(p) = \chi(0) + A^{\mu\nu} p_\mu p_\nu + \text{convergent}$, $A^{\mu\nu} = \frac{1}{2} \partial_\mu \partial_\nu \chi(0)$. We have:

$$A^{\mu\nu} = c_2 \eta^{\mu\nu} + a^{\mu\nu}$$

c_2 is the log divergent wave function renormalization counterterm; $a^{\mu\nu}$ is a finite LIV. The on-shell condition is:

$$p^2 - m^2 - a^{\mu\nu} p_\mu p_\nu = 0$$

If spatial rotational invariance is preserved, the nonzero components of the matrix a are:

$$a^{00} = a_0; \quad a^{ii} = -a_1$$

So the maximum attainable velocity for this particle will be:

$$c_m = \sqrt{\frac{1-a_1}{1-a_0}} \sim 1 - (a_1 - a_0)/2 \quad (1)$$

By doing explicit computations for all particles in the SM, we get definite predictions for the LIV, assuming a particular regulator R . However, the dependence on R amounts to a multiplicative factor. So ratios of LIV's are uniquely determined. Explicit computations are simplified by using Dimensional Regularization as explained below.

LIV DIMENSIONAL REGULARIZATION

We generalize dimensional regularization to a d dimensional space with an arbitrary constant metric $g_{\mu\nu}$. We work with a positive definite metric first and then Wick rotate. We will illustrate the procedure with an example. Here $g = \det(g_{\mu\nu})$ and $\Delta > 0$.

$$\begin{aligned} & \frac{1}{\sqrt{g}} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2 + \Delta)^n} = \\ & \frac{1}{\sqrt{g}\Gamma(n)} \int_0^\infty dt t^{n-1} \int \frac{d^d k}{(2\pi)^d} k_\mu k_\nu e^{-t(g^{\alpha\beta} k_\alpha k_\beta + \Delta)} = \\ & \frac{1}{(4\pi)^{d/2}} \frac{g_{\mu\nu}}{2} \frac{\Gamma(n-1-d/2)}{\Gamma(n)} \frac{1}{\Delta^{n-1-d/2}} \end{aligned} \quad (2)$$

In the same manner, after Wick rotation, we obtain Appendix A4 of [20]. These definitions preserve gauge invariance, because the integration measure is invariant under shifts. To get a LIV measure, we assume that

$$g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0} \eta^{v 0} \text{Res}_{\varepsilon=0} \quad (3)$$

where $\varepsilon = 2 - \frac{d}{2}$ and $\text{Res}_{\varepsilon=0}$ is the residue of the pole at $\varepsilon = 0$. A formerly divergent integral will have a pole at $\varepsilon = 0$, so when we take the physical limit, $\varepsilon \rightarrow 0$, the answer will contain a LIV term.

That is, LIV dimensional regularization consists in: 1) Calculating the d-dimensional integrals using a general metric $g_{\mu\nu}$. 2) Gamma matrix algebra is generalized to a general metric $g_{\mu\nu}$. 3) At the end of the calculation, replace $g^{\mu\nu} = \eta^{\mu\nu} + (4\pi)^2 \alpha \eta^{\mu 0} \eta^{v 0} \text{Res}_{\varepsilon=0}$ and then take the limit $\varepsilon \rightarrow 0$.

To define the counterterms, we used the minimal subtraction scheme(MSS); that is we subtract the poles in ε from the 1PI graphs.

LIV Dimensional Regularization reinforces our claim that these tiny LIV's originates in Quantum Gravity. In fact the sole change of the metric of space time is a correction of order ε to the Minkowsky metric and this is the source of the effects studied above. Quantum Gravity is the strongest candidate to produce such effects because the gravitational field is precisely the metric of space-time and tiny LIV modifications to the flat Minkowsky metric may be produced by quantum fluctuations.

LIV in the Standard Model

We follow [22, 23, 8] and use LIV Dimensional Regularization.

Photons

The LIV photon self-energy in the SM is:

$$L\Pi^{\mu\nu}(q) = -\frac{23}{3}e^2 \alpha q_\alpha q_\beta (\eta^{\alpha\beta} \delta_0^\mu \delta_0^\nu + \eta^{\mu\nu} \delta_0^\alpha \delta_0^\beta - \eta^{\nu\beta} \delta_0^\mu \delta_0^\alpha - \eta^{\mu\alpha} \delta_0^\nu \delta_0^\beta) \quad (4)$$

It follows that the maximal attainable velocity is

$$c_\gamma = 1 - \frac{23}{6}e^2 \alpha \quad (5)$$

We have included coupling to quarks and charged leptons as well as 3 generations and color.

Fermions

In the SM, the fermion self-energy is given by:

$$(4\pi^2)\Sigma(q) = -\frac{1}{\varepsilon} \not{q} \sum_{graphs} (|c_V + c_A|^2 P_L + |c_V - c_A|^2 P_R) + finite \quad (6)$$

where the fermion-gauge boson vertex is:

$$i\gamma^k (c_V - c_A \gamma^5) \quad (7)$$

and $P_L(P_R)$ are the L(R) helicity projectors.

Therefore

$$L\Sigma(q) = \frac{\alpha}{2} q_0 \gamma^0 \sum_{graphs} (|c_V + c_A|^2 P_L + |c_V - c_A|^2 P_R) \quad (8)$$

We apply this last result to neutrinos and charged leptons below.

Neutrinos: The maximal attainable velocity is

$$c_V = 1 - (3 + \tan^2 \theta_w) \frac{g^2 \alpha}{8} \quad (9)$$

In this scenario, we predict that neutrinos [18] emitted simultaneously with photons in gamma ray bursts will not arrive simultaneously to Earth. The time delay during a flight from a source situated at a distance D will be of the order of $(5 \times 10^{-23}) D/c \sim 5 \times 10^{-6}$ s, assuming $D = 10^{10}$ light-years. No dependence of the time delay on the energy of high energy photons or neutrinos should be observed (contrast with [1]). Photons will arrive earlier since $\alpha < 0$ (See below). These predictions could be tested in the next generation of neutrino detectors [19].

Using R_ξ -gauges we have checked that the LIV is gauge invariant. The gauge parameter affects the Lorentz invariant part only.

Electron self-energy in the Weinberg-Salam model. Birrefringence:

Define: $e_L = \frac{1-\gamma^5}{2} e$, $e_R = \frac{1+\gamma^5}{2} e$, where e is the electron field. We get

$$c_L = 1 - \left(\frac{g^2}{\cos^2 \theta_w} (\sin^2 \theta_w - 1/2)^2 + e^2 + g^2/2 \right) \frac{\alpha}{2}; \quad (10)$$

$$c_R = 1 - \left(e^2 + \frac{g^2 \sin^4 \theta_w}{\cos^2 \theta_w} \right) \frac{\alpha}{2} \quad (11)$$

The difference in maximal speed for the left and right helicities is $\sim (5 \times 10^{-24})$.

MESONS AND BARYONS

In order to calculate the maximal attainable velocity of hadrons, we will use effective lagrangians.

We use the results of [24, 25] for the wave function renormalization of pions and nucleons in the chiral lagrangian and Heavy Baryon Chiral Perturbation Theory. They get:

$$Z_\pi^{-1} = 1 - \frac{4m_\pi^2}{3(4\pi)^2 F^2} \frac{1}{\epsilon} + finite \quad (12)$$

$$Z_N^{-1} = 1 - \frac{9g_A^2 m_\pi^2}{4(4\pi)^2 F^2} \frac{1}{\epsilon} + finite \quad (13)$$

Here, m_π is the renormalized pion mass, F is the renormalized decay constant of pions and g_A is the axial vector coupling constant, in the chiral limit.

Using the LIV metric, we can read off the maximal attainable velocities for pions and nucleons:

$$\begin{aligned} c_\pi &= 1 + \frac{2m_\pi^2\alpha}{3F^2} \\ c_N &= 1 + \frac{9m_\pi^2g_A^2\alpha}{8F^2} \end{aligned} \quad (14)$$

REACTION THRESHOLDS AND BOUNDS

Knowing the LIV for nucleons, pions, photons and electrons, we proceed to study the reactions involved in the GZK cutoff. We follow the discussion in [5, 13, 8].

Photo-Pion Production $\gamma + p \rightarrow p + \pi$

Let us begin with the photo-pion production $\gamma + p \rightarrow p + \pi$. Considering the corrections provided in the dispersion relation (14) for pions and nucleons, we note that, for the photo-pion production to proceed, the following condition must be satisfied

$$2\delta c E_\pi^2 + 4E_\pi\omega \geq \frac{m_\pi^2(2m_p + m_\pi)}{m_p + m_\pi}. \quad (15)$$

where E_π is the produced pion energy and $\delta c = c_p - c_\pi$.

Pair Creation $\gamma + p \rightarrow p + e^+ + e^-$

Pair creation, $\gamma + p \rightarrow p + e^+ + e^-$, is greatly abundant in the sector previous to the GZK limit. When the dispersion relations for fermions are considered for both protons and electrons, it is possible to find

$$\delta c \frac{m_e}{m_p} E^2 + E\omega \geq m_e(m_p + m_e), \quad (16)$$

where E is the incident proton energy and $\delta c = c_p - c_e$.

Combining the two reactions and the standard values, $m_\pi = 139\text{MeV}$, $g_A = 1.26$, $F = 92.4\text{MeV}$, we get an upper and lower bound on α

$$2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24} \quad (17)$$

First of all, we notice that $\alpha < 0$, in order to suppress the photopion production, thus removing the GZK cutoff. This implies that photons are the fastest particles and they arrive before neutrinos coming from the same source of GRB. Moreover, photons become unstable. They decay in a electron positron pair above an energy E_0 [5]. See below.

Since $c_{\text{photon}} > c_{\text{proton}}$, the strong bound of [29] is avoided: Proton is stable under Cerenkov radiation in vacuum.

If no GZK anomaly is confirmed in future experimental observations, then we should state a stronger bound for the difference $c_\pi - c_p$. Using the same assumptions to set the restriction (17) when the primordial proton reference energy is $E_{\text{ref}} = 2 \times 10^{20}$ eV, it is possible to find

$$|c_\pi - c_p| < 2.3 \times 10^{-23}. \quad (18)$$

In terms of α , this last bound may be read as

$$|\alpha| < 9.1 \times 10^{-24}, \quad (19)$$

Photon unstability

It has been pointed out in [29, 5] that if $c_{photon} > c_{electron}$ then the process $\gamma \rightarrow e^+ + e^-$ is allowed above an energy E_0 :

$$E_0 = m_e \sqrt{\frac{2}{\delta c}} \quad (20)$$

where $\delta c = c_\gamma - c_e$.

In our case, we have:

$$\delta c_L = -\alpha \left(\frac{23}{6} e^2 - \left(\frac{g^2}{\cos^2 \theta_w} (\sin^2 \theta_w - 1/2)^2 + e^2 + g^2/2 \right) / 2 \right) \quad (21)$$

$$\delta c_R = -\alpha \left(\frac{23}{6} e^2 - \left(e^2 + \frac{g^2 \sin^4 \theta_w}{\cos^2 \theta_w} \right) / 2 \right) \quad (22)$$

Therefore, with

$$\begin{aligned} EL_0 &= 2.3 \times 10^8 \text{ GeV} \\ ER_0 &= 1.9 \times 10^8 \text{ GeV} \end{aligned} \quad (23)$$

So, we should not detect photons with energies above $2.3 \times 10^8 \text{ GeV}$

Neutral pion Stability

Following [5] we study the main decay process of neutral pion $\pi_0 \rightarrow \gamma + \gamma$. This becomes possible if $c_\gamma > c_\pi$ and above an energy

$$E_\pi = \frac{m_\pi}{\sqrt{2(c_\gamma - c_\pi)}} \quad (24)$$

Using the bound $c_\gamma - c_\pi < 10^{-22}$ obtained in [30], we get

$$|\alpha| < 5.4 \times 10^{-23} \quad (25)$$

In our numerical estimates we have chosen $\alpha = -5 \times 10^{-23}$.

We get $E_\pi = 10^{19} \text{ eV}$. Therefore we expect that neutral pions above this energy are stable, so they could be a primary component of UHECR. Photons will be unstable above this energy by the same mechanism. Notice however that photons are unstable at a lower energy due to electron-positron pair creation (23).

CONCLUSIONS

In this paper we have computed the LIV induced by Quantum Gravity on Baryons and Mesons, using the Chiral Lagrangian approach. This permitted to fix that $\alpha < 0$, in order to explain the GZK anomaly. Studying several available processes, we found bounds on α :

From pair creation and absence of photopion creation: $2.2 \times 10^{-21} > -\alpha > 1.3 \times 10^{-24}$.

From pion stability and the most stringent experimental bound found in [30]: $|\alpha| < 5.4 \times 10^{-23}$.

Then, several predictions are obtained: Photons are unstable above an energy $2.3 \times 10^8 \text{ GeV}$.

Neutral pions are stable above an energy $E_\pi = 10^{19} \text{ eV}$; so they could be a primary component of UHECR, thus evading the GZK cutoff.

Moreover, in time of flight experiments, photons will arrive before neutrinos, assuming that they were emitted simultaneously at the source. No energy dependence of the time delay should be observed. The time delay during a flight from a source situated at a distance D will be of the order of $(5 \times 10^{-23})D/c \sim 5 \times 10^{-6} \text{ s}$, assuming $D = 10^{10}$ light-years.

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